Q.1: A number is selected at random from first thirty natural numbers. What is the probability that it is a multiple of either 3 or 13?

Soln.- The first thirty natural numbers are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

E = Event that number is multiple of either 3 or 13.

No. which are multiple of 3 in S = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

No. which are multiple of 13 in S = 13, 26

\[ P(E) = \frac{\text{No. multiple of 3}}{30} + \frac{\text{No. multiple of 13}}{30} \]

\[ = \frac{10}{30} + \frac{2}{30} \]

\[ = \frac{1}{3} + \frac{1}{15} \]

\[ = \frac{6}{15} = \frac{2}{5} \]

Q.2: Let A and B be independent events with \( P(A) = \frac{1}{4} \) and \( P(A \cup B) = 2P(B) - P(A) \). Find

a) \( P(B) \)

b) \( P\left(\frac{A}{B}\right) \)

c) \( P\left(\frac{B}{A}\right) \)

Soln:- a) \( P(A) = \frac{1}{4} \)

\( P(A \cup B) = 2P(B) - P(A) \)

\( \therefore \) A and B are independent events

So, \( P(A \cup B) = P(A) \cdot P(B) \)

\[ P(A \cup B) = P(A) + P(B) - P(\overline{A} \overline{B}) \]
\[ = P(A) + P(B) - P(A) \cdot P(B) \]
\[ = \frac{1}{4} + P(B) - \frac{1}{4} \cdot P(B) \quad (\because P(A) = \frac{1}{4}) \]
\[ = \frac{1}{4} + P(B) \left(1 - \frac{1}{4}\right) \]
\[ P(A \cup B) = \frac{1}{4} + \frac{3}{4} \cdot P(B) - (1) \]

**Given:**
\[ P(A \cup B) = 2P(B) - P(A) - (2) \]

From eqn 1 and 2:
\[ \frac{1}{4} + \frac{3}{4} \cdot P(B) = 2P(B) - P(A) \]
\[ \frac{1}{4} + \frac{3}{4} \cdot P(B) = 2P(B) - \frac{1}{4} \]
\[ \frac{1}{2} = 2P(B) - \frac{3}{4} \cdot P(B) \]
\[ \frac{5}{4} \cdot P(B) = \frac{1}{2} \]
\[ P(B) = \frac{4}{5} \times \frac{1}{2} = \frac{2}{5} \]

**b)** \[ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A) = \frac{1}{4} \]

**c)** \[ P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B) - P(A)}{P(A)} \]
\[ = \frac{1}{2} \cdot P(B) \]
\[ = \frac{1}{2} \times \frac{2}{5} \]
\[ = \frac{1}{5} \]

**Q3:** Five men in a company of twenty are graduates. If three men are picked out of twenty at random, what is the probability that
a) they are all graduates
b) at least one is a graduate.
Solution:

Total no. of men in a company = 20
No. of graduates in company = 5
No. of person to be picked at random = 3
Sample Space = 20C3

a) $E_1$ = Event of taking three men in which all are graduates.

Way to pick three men which are graduates out of 5 = 5C3

$$P(E_1) = \frac{5C3}{20C3} = \frac{1}{124}$$

b) $E_2$ = Event of taking at least one graduate.

$$P(E_2) = \text{at least one graduate + at least two graduates + all three graduates}$$

$$= \frac{5C_2 \times 15C_2}{20C_3} + \frac{5C_2 \times 15C_1}{20C_3} + \frac{5C_3}{20C_3}$$

$$= \frac{137}{228}$$

8.4: There are two urns. First urn contains 2W and 2B balls, second urn contain 3W and 4B balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is found black, find the probability that the urn chosen was first.

Solution:

Let $E_1$ be the event of choosing Urn 1.

$E_2$ be the event of choosing Urn 2.

Let $E$ be the event of choosing black ball.

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(E/E_1) = \text{Probability that the drawn ball is black and that the ball is from Urn 1.}$$

$$= \frac{2/4}{1/2} = \frac{1}{2}$$
\[ P(E/E_2) = \text{probability that the drawn ball is black and that the ball is from Urn 2}. \]

\[ P(E/E_2) = \frac{4}{7} \]

Probability of drawing a ball is black and it is drawn from Urn 1 = \( P(E/E) \)

\[ P(E/E) = \frac{P(E/E) \cdot P(E)}{P(E/E) \cdot P(E) + P(E/E_2) \cdot P(E_2)} \]

\[ = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{4}{9} \times \frac{1}{2}} \]

\[ P(E/E) = \frac{7}{15} \]

Q: What is the probability of getting at least one 6 in a single throw of three unbiased dice?

Sol.: No. of dice = 3

Possible outcomes = \( 6 \times 6 \times 6 = 216 \)

\( E = \text{Event of getting at least one 6}. \)

\[ P(E) = \text{at least one six} + \text{two sixes} + \text{three sixes} \]

\[ = \frac{25}{216} + \frac{15}{216} + \frac{1}{216} = \frac{41}{216} \]

Q: A picnic is arranged to be held on a particular day. The weather forecast says that there is 80% chance of rain on that day. If it rains, the probability of a good picnic is 0.3 and if it does not rain, the probability is 0.9.

What is the probability that the picnic will be good?
Solve:

Chance of rain = 80% = \frac{80}{100} = \frac{8}{10}

Chance of not raining = 20% = 1 - \frac{8}{10} = \frac{2}{10} = \frac{1}{5}

If it rains the probability of a good picnic = 0.3 = \frac{3}{10}

If it doesn't rain the probability of a good picnic = 0.9 = \frac{9}{10}

E = Event of a good picnic

P(E) = \text{It is rain} \cdot \text{probability of good picnic} + \text{It is doesn't rain} \cdot \text{probability of good picnic}

= \frac{8}{10} \times \frac{3}{10} + \frac{1}{5} \times \frac{9}{10}

= \frac{24}{100} + \frac{9}{50}

= \frac{21}{50}

Q. 7: Two digits are selected at random from the digits 1 through 9 (a) If the sum is odd, what is the probability that 2 is one of the numbers selected.

(b) If 2 is one of the digits selected, what is the probability that the sum is odd?

Solve:

Given digits are: 1, 2, 3, 4, 5, 6, 7, 8, 9

Odd numbers = 1, 3, 5, 7, 9

Even numbers = 2, 4, 6, 8

To get sum as odd = \text{Even no. + Odd no.}

9) E = Event of getting sum as odd.
\[ E_1 = \text{Event of getting sum as odd if the even no is } 2. \]
\[ E_2 = \text{Event of getting sum as odd if the even no is } 4. \]
\[ E_3 = \text{Event of getting sum as odd if the even no is } 6. \]
\[ E_4 = \text{Event of getting sum as odd if the even no is } 8. \]

\[ P(E / E_i) = P(E / E_2) = P(E / E_3) = P(E / E_4) = \frac{1 \times 5 \times 1}{36} = \frac{5}{36} \]

Now when \( E_1, E_2, E_3, E_4 \) are events of getting no 2, 4, 6, 8 respectively, \( P(E_1 / E) \) probability of getting one no is 2 if the sum is odd.

\[ P(E_1 / E) = \frac{P(E_1 / E) \cdot P(E)}{P(E_1) \cdot P(E) + P(E_2) \cdot P(E_2) + P(E_3) \cdot P(E_3) + P(E_4) \cdot P(E_4)} \]

\[ P(E_1 / E) = \frac{5/36 \times 1}{5/36 \times \frac{1}{9} + 5/36 \times \frac{1}{9} + 5/36 \times \frac{1}{9} + 5/36 \times \frac{1}{9}} \]

\[ = \frac{5/36 \times 1/9}{4 \times 5/36 \times 1/9} \]

\[ = \frac{1}{9} \]

b) given that one number is 2, if even no is 2 then
\( E = \text{Event of getting sum as odd} \)
\[ P(E) = \frac{\text{one odd no} \neq 2}{\text{all collection times}} = \frac{5(1 \times 1)}{9(2)} = \frac{5}{36} \]