Chapter -1

Numerical Methods -1

1. Using the Taylor’s method, find the third order approximate solution at x=0.4 of the problem \( \frac{dy}{dx} = x^2y + 1 \), with y(0)=0. Consider up to fourth degree terms.

2. Solve the differential equation \( \frac{dy}{dx} = -xy^2 \) under the initial condition y(0) = 2, by using the modified Euler’s method, at the points x = 0.1 and x = 0.2. Take the step size h = 0.1 and carry out two modifications at each step.

3. Given \( \frac{dy}{dx} = xy + y^2 \), y(0)=1.1169, y(0.2)=1.2773, y(0.3)=1.5049, find y(0.4) correct to three decimal places, using Milne’s predictor-corrector method. Apply the corrector formula twice.

4. Solve \( \frac{dy}{dx} = x + y \), x=0, y=1 at x = 0.2 using Runge-Kutta method taking h=0.2.

5. Using Euler’s modified method, solve for y at x=0.1, if \( \frac{dy}{dx} = \frac{y-x}{y+x} \), y(0)=1. Carry out three modifications.

6. Given \( \frac{dy}{dx} = (1+y)x^2 \) and y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979, determine y(1.4) by Adams-Bash forth method.

7. Apply Runge-Kutta method of order four to compute y(2.0), given \( 10 \frac{dy}{dx} = x^2 + y^2 \), y(0) = 1, taking h = 0.1
8. The following table gives the solution of \( \frac{dy}{dx} = x - y^2 \), find the value of \( y \) at \( x = 0.8 \), using Milne’s predictor and corrector formulae.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0</td>
<td>0.02</td>
<td>0.07</td>
<td>0.17</td>
</tr>
</tbody>
</table>

9. Given \( \frac{dy}{dx} + y - x^2 = 0 \), \( y(0) = 1 \), \( y(0.1) = 0.9052 \), \( y(0.2) = 0.8213 \). Find the values of \( y(0.3) \), \( y(0.4) \) correct to four decimal places using modified Euler’s method.

10. Using Taylor’s series method find \( y(4.1) \) given \( \frac{dy}{dx} = \frac{1}{x^3 + y} \) and \( y(4) = 4 \).

Chapter -2

Numerical Methods -2

1. Solve: \( \frac{dy}{dx} = \frac{(y^2 - x^2)}{y^2 + x^2} \) with \( y(0) = 1 \), find \( y \) at \( x = 0.2 \) using Runge –Kutta method of \( 4^{th} \) order taking step-length \( h = 0.2 \) accurate up to \( 4^{th} \) decimal places.

2. Using Runge –Kutta method of order 4 to compute \( y(0.2) \) for the equation \( y' = y - \frac{2x}{y} \), \( y(0) = 1 \) (take \( h = 0.2 \))

3. Use Picard’s method to obtain the third approximation to the solution of \( \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 6y = 0 \) given that when \( x = 0 \), \( y = 1 \), \( \frac{dy}{dx} = 0.1 \).

4. Find the second approximation for the solution of the following system of equations by Picard ‘s method \( \frac{dx}{dt} = (x + y)t \), \( \frac{dy}{dt} = (x - t)y \), \( x = 0, y = 1 \) at \( t = 0 \).

5. Using Milne’s method, obtain an approximate solution at the point \( x = 0.4 \) of the problem \( \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 6y = 0 \), \( y'(0) = 0.1 \). Given that \( y(0.1) = 1.03995 \), \( y(0.2) = 1.138036 \), \( y(0.3) = 1.29865 \), \( y'(0.1) = 0.6955 \), \( y'(0.2) = 1.258 \), \( y'(0.3) = 1.873 \).

6. Applying Milne’s predictor and corrector formulae compute \( y(0.8) \) given that \( y \) satisfies the equation \( y'' = 2yy' \) and \( y \) and \( y' \) are governed by the following values \( y(0) = 0 \),
\[ y(0.2) = 0.2027, \ y(0.4) = 0.4228, \ y(0.6) = 0.6841, \]
\[ y(0) = 1, \ y'(0.2) = 1.041, \ y'(0.4) = 1.179, \ y'(0.6) = 1.468. \]

Apply corrector formula twice.

7. Employing the Picard’s method, obtain the second order approximate solution of the following problem at \( x = 0.2 \). \( \frac{dy}{dx} = x + yz, \frac{dz}{dx} = y + zx, y(0) = 1, z(0) = -1. \)

8. Using Runge-Kutta method, find the solution at \( x = 0.1 \) of the differential equation \( \frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} - 2xy = 1 \) under the conditions \( y(0) = 1, \ y'(0) = 0. \) Take step length \( h = 0.1. \)

9. Using Runge-Kutta method, solve the following D.E at \( x = 0.1 \) under the given condition \( \frac{d^2y}{dx^2} = x^3(y + \frac{dy}{dx}), \ y(0) = 1, \ y'(0) = 0.5. \)

10. Apply Picard’s method to compute \( y(1.1) \) from the second approximation to the solution of the equation \( y'' + y^2y' = x^3 \) given that \( y \) and \( y' \) have the value 1 at \( x = 1. \)
Chapter -3

Complex variables-1

1. Derive Cauchy-Riemann equations in Cartesian form.

2. Derive Cauchy-Riemann equations in polar form.

3. Determine the analytic function \( f(z) = u + iv \), if \( u - v = \frac{\sin 2x}{\sinh 2y - \cos 2x} \)

4. If \( f(z) = u + iv \) is analytic, prove that \( \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2 \)

5. Determine the analytic function whose real part is \( \cos x \cosh y \).

6. Find the analytic function \( f(z) \) given \( u = e^{-z} \left\{ (x^2 - y^2) \cos y + 2xy \sin y \right\} \)

7. If \( f(z) \) is a regular function of \( z \) show that

\[
\left\{ \frac{\partial}{\partial x} |f(z)| \right\}^2 + \left\{ \frac{\partial}{\partial y} |f(z)| \right\}^2 = |f'(z)|^2
\]

8. An electro static field in the xy plane is given by the potential function \( \phi = 3x^2 y - y^3 \). Find the stream function.

9. If \( w = \phi + i \psi \) represents the complex potential for an electric field and

\[
\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}
\]

Find the scalar potential \( \phi \).

10. If \( f(z) = u(r, \theta) + iv(r, \theta) \) is an analytic function, show that \( u \) and \( v \) satisfy the equation

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0
\]
Chapter -4

Complex variables-2

1. Find the bilinear transformation which maps the points \( z = 1,1,-1 \) onto the points \( w = i,0,-i \).

2. Expand \( f(z) = \frac{1}{(z-1)(z-2)} \) in the region i) \( |z| < 1 \) and ii) \( 1 < |z| < 2 \).

3. Evaluate \( \int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} \, dz \) where \( C: |z| = 3 \).

4. Evaluate \( \int_C \frac{e^{\pi z}}{(2z-i)^3} \, dz \) where \( C \) is the circle \( |z| = 1 \).

5. State and prove Cauchy’s theorem.

6. Evaluate \( \int_C \frac{\sin^6 z}{(z-z_0)^3} \, dz \) where \( C \) is the circle \( |z| = 1 \).

7. Find the bilinear transformation which maps the points \( z = 0,1,\infty \) onto the points \( W = -5, -1, 3 \) respectively. Also find the invariant points.

8. Evaluate \( \int_C |z|^2 \, dz \) where \( C \) is a square with vertices (0,0),(1,0),(1,1) and (0,1).

9. Verify Cauchy’s theorem for the function \( f(z) = z e^{-z} \) over the unit circle with origin as the centre.

10. If \( C \) is a circle with centre ‘\( a \)’ and radius ‘\( r \)’ then show that

   \( i \) \( \int_C \frac{1}{z-a} \, dz = 2\pi i \)

   \( ii \) \( \int_C (z-a)^n \, dz = 0 \) if \( n \neq -1 \)
Chapter -5

Special Functions

1. Express \( f(x) = (x + 1)(x + 2)(x + 3) \) in terms of Legendre polynomial.

2. State Rodrigue’s formula for Legendre polynomial and obtain the expression for \( P_4(x) \) from it. Verify the property of Legendre polynomials in respect of \( P_4(x) \) and also find \( \int_{-1}^{1} x^3 P_4(x) \, dx \).

3. Verify that \( y = x J_n(x) \) is a solution of the differential equation
   \[ x^2 y'' - xy' + (1 + x^2 - n^2)y = 0. \]

4. S.T \( \frac{d}{dx} [x^n J_n(x)] = x^n J_{n-1}(x) \).

5. Express \( J_5(x) \) in terms of \( J_0(x) \) and \( J_1(x) \).

6. Express the polynomial \( x^3 + x^2 + x + 1 \) in terms of Legendre’s polynomial.

7. Reduce the differential equation \( x^2 y'' + xy' + (k^2 x^2 - n^2)y = 0 \) into Bessel’s form and write complete solutions for \( n \).

8. Obtain \( P_3(x) \) from Rodrigue’s formula and the same satisfies the Legendre’s equation in the standard form.

9. S.T \( \int_{0}^{x} x^{-n} J_{n+1}(x) \, dx = \frac{1}{2^n} \frac{1}{\Gamma(n+1)} \frac{J_n(x)}{x^n} \).

10. State and prove Bessel’s differential equation of order ‘\( n \)’.
1. From five positive and seven negative numbers, five numbers are chosen at random and multiplied. What is the probability that the product is a (i) negative number and (ii) positive number?

2. If A and B are two events with \( P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{4} \), find \( P(A/B), P(B/A), P(A/B^c), P(B/A^c) \) and \( P(A/B) \).

3. In a certain college, 4% of boy students and 1% of girl students are taller than 1.8 m. Furthermore, 60% of the students are girls. If a student is selected at random and is found taller than 1.8 m, what is the probability that the student is a girl?

4. The probability of four persons A, B, C, D hitting targets are respectively 1/2, 1/3, 1/4, 1/5. What is the probability that the target is hit by at least one person if all are hit simultaneously?

5. i) State addition law of probability for any two events A and B. ii) Two different digits from 1 to 9 are selected. What is the probability that the sum of the two selected digits is odd if ‘2’ is one of the digits selected?

6. Three machines A, B, C produce 50%, 30%, 20% of the items. The percentage of defective items are 3, 4, 5 respectively. If the item selected is defective, what is the probability that it is from machine A? Also find the total probability that an item is defective.

7. State the axioms of probability. For any two events A and B, prove that \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \).

8. A bag contains 10 white balls and 3 red balls while another bag contains 3 white balls and 5 red balls. Two balls are drawn at random from the first bag and put in the second bag and then a ball is drawn at random from the second bag. What is the probability that it is a white ball?

9. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production. Out of these 5%, 4%, 3% and 2% respectively
are defective. A bolt is drawn at random from the production and is found to be defective. Find the probability that it was manufactured by A or D.

10. A pair of dice is tossed twice. Find the probability of scoring 7 points (a) once, (b) at least once and (c) twice.

Chapter -7
PROBABILITY THEORY- 2

1. A random variable $x$ has the density function $P(x) = \begin{cases} k x^2, & 0 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$

Evaluate $k$ and find

i) $p (x \leq 1)$
ii) $p (1 \leq x \leq 2)$
iii) $p (x \leq 2)$
iv) $p (x > 1)$
v) $p (x > 2)$

2. Obtain the mean and standard deviation of binomial distribution

3. Obtain the mean and standard deviation of Poisson distribution

4. The probability density function of a variate $X$ is

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>K</td>
<td>3K</td>
<td>5K</td>
<td>7K</td>
<td>9K</td>
<td>11K</td>
<td>13K</td>
</tr>
</tbody>
</table>

Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$

5. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution

6. A die is thrown 8 times. Find the probability that 3 falls,
   i) exactly 2 times
   ii) at least once
   iii) at the most 7 times

7. In a certain town the duration of shower has mean 5 minutes. What is the probability that shower will last for
   i) 10 minutes or more
   ii) less than 10 minutes
   iii) between 10 and 12 minutes

8. Obtain the mean and variance of exponential distribution

9. Fit a Poisson distribution to the following data and calculate the theoretical frequencies
10. In a test of 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours. Estimate the number of bulbs likely to burn for
i) more than 2150 hours  ii) less than 1950 hours  and iii) more than 1920 hours and but less than 2160 hours

Chapter -8
SAMPLING THEORY

1. A biased coin is tossed 500 times and head turns up 120 times. Find the 95% confidence limits for the proportion of heads turning up infinitely many tosses. (given that \( Z_c = 1.96 \)).

2. A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 (in appropriate unit) Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use \( t_{0.05(11)} = 2.201 \).

3. A random sample of 400 items chosen from an infinite population is found to have a mean of 82 and a standard deviation of 18. Find the 95% confidence limits for the mean of the population from which the sample is drawn.

4. In the past, a machine has produced washers having a thickness of 0.50mm. To determine whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is found as 0.53mm with standard deviation 0.03mm. Test the hypothesis that the machines is in proper working order, using a level of significance of (i) 0.05 and (ii) 0.01.

5. Genetic theory states that children having one parent of blood type M and the other of blood type N will always be one of the three types M, MN, N and that the proportions of these types will on an average be 1:2:1. A report states that out of 300 children having one M parent and one N parent, 30% were found to be of type M, 45% of type MN and the reminder of type N. Use Chi-Square distribution to test the theory.

6. A die is thrown 60 times and the frequency distribution for the number appearing on the face X is given by the following table:

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>122</td>
<td>60</td>
<td>15</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>X</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Frequency</td>
<td>15</td>
<td>6</td>
<td>4</td>
<td>7</td>
<td>11</td>
</tr>
</tbody>
</table>

Test the hypothesis that the die is unbiased. (Given that $\chi^2_{0.05}(5) = 11.07$ and $\chi^2_{0.01}(5) = 15.09$

7. Define the following:
   i) Transient state  
   ii) Recurrent state  
   iii) Absorbing state of a Markov chain.

8. Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find the associated unique fixed probability vector.

9. The joint probability distribution table for two random variables $x$ and $y$ are as follows:

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>-2</td>
<td>-1</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

   Determine the marginal probability distributions of $X$ and $Y$. Also compute
   i) Expectations of $x$, $y$ and $xy$  
   ii) S.D of $x$, $y$  
   iii) Covariance of $x$ and $y$.

10. A gambler’s luck follows a pattern. If he wins a game, the probability of winning the next game is 0.6. However if he loses a game, the probability of losing the next game is 0.7. There is an even chance of the gambler winning the first game. If so:
   i) What is the probability of he winning the second game?  
   ii) What is the probability of he winning the third game?  
   iii) in the long run, how often will he win?